J. Fluid Mech. (1969), *vol.* 35, *part* 3, *pp.* 549–559
Printed in Great Britain

An analytical and experimental investigation of the trajectories of particles entrained by the gas flow in nozzles

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(Received 1 March 1968)

Among the parameters which determine the erosion damage sustained by the walls of a nozzle in which a mixture of gas and particles is flowing, is the angle between the direction of the particle flow and the wall surface at the moment of impact. In this work an approximate analytical solution is made for a number of gas particle flows to determine broadly the features on which particle trajectory depends and some experimental results are given which confirm the theoretical computations. It is shown that the divergent region of a conical nozzle is unlikely to suffer a severe particle attack but that for parallel flow convergent–divergent nozzles the convex region near the exit may be affected. The choke, on the other hand, is most susceptible to particle attack even by fairly small particles. It may be said, in general, that any particle which enters the choke section with a velocity which, in the absence of effects from the gas would allow the particle to strike the choke wall, will in fact hit the wall at some point along the length of the choke.

1. Introduction

This investigation was part of a study of the erosion damage sustained by nozzles passing a mixture of gas and particles with particular reference to that part of the erosion in rocket motor tail nozzles which can be attributed to the action of particles produced in the combustion chamber. The erosion suffered by the nozzle wall at a given cross-section in the flow has been shown to depend on the properties of the wall material, the total mass of impinging particles, the particle speed and the angle of attack. A one-dimensional treatment, used to find particle velocity at a given cross-section in a nozzle (Neilson & Gilchrist 1968), indicated that this could be determined by assuming that the gas conditions throughout the nozzle were unaffected by the presence of particles, provided the proportion of particles present was not too large. This assumption greatly simplifies the procedure needed to determine the path taken by the particle and, if during the excursion through the nozzle the particle does strike the nozzle wall, the angle of attack may then be obtained. This simplification is especially

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important since, of necessity, two- or three-dimensional features of the flow must be taken into account.

To determine particle trajectories in an axi-symmetric nozzle it is first necessary to obtain the radial and longitudinal gas components of velocity at all points in the nozzle. Since this velocity field is the same in every longitudinal plane passing through the axis of the nozzle, the problem may be reduced to finding the particle path in a two-dimensional flow field with a known gas velocity distribution.

2. Application of the drag equation in a two-dimensional flow field

The gas flow field in the supersonic régime of a nozzle may be obtained by the method of characteristics. For gas velocity components in the region of the throat the method for transonic flow given by Oswatitsch & Rothstein (1949) may be used and this method may also be extended to give an estimate of the velocity components in the choke. At any given value of x, figure 1 (*b*), the components of the gas velocity (U_g) in the x- and y-directions (U_{gx} and U_{gy} respectively) may be expressed as

$$U_{gx} = A + By^2 + Cy^4 \tag{1}$$

and

$$U_{gy} = Dy + Ey^3, \tag{2}$$

where A, B, C, D and E are known constants.

Consider now a point (x, y) in a two-dimensional flow field where the particle and gas velocities are known, figure 1 (*a*). The total force (F) acting on the particle is

$$F = C_d \tfrac{1}{2}\rho_g V_r^2 A_p, \tag{3}$$

where ρ_g is the gas density, V_r the relative velocity between the gas and the particle, A_p the particle cross-section area and C_d the drag coefficient. For a sphere Gilbert, Davis & Altman (1955) have shown that

$$C_d = f(24/Re), \tag{4}$$

where

$$f = \frac{28Re^{-0\cdot85} + 0\cdot48}{24\,Re^{-1\cdot0}} \tag{5}$$

and Re is the Reynolds number.

Thus the total force acting on the particle is

$$F = f(24/Re)\,\tfrac{1}{2}\rho_g V_r^2 A_p. \tag{6}$$

Thus the force component in the x-direction is

$$mU_{px}\frac{dU_{px}}{dx} = f\frac{24}{Re}\,\tfrac{1}{2}\rho_g V_r^2 A_p \cos\theta,$$

where m is the particle mass, U_{px} the component of the particle velocity in the x-direction and θ the angle between the relative velocity vector and the x-direction. This relationship may be re-arranged to give

$$U_{px}(dU_{px}/dx) = \alpha f(U_{gx} - U_{px}), \tag{7}$$

where $$\alpha = 18\mu_g/\rho_p d_p^2, \tag{8}$$
μ_g is the coefficient of viscosity for the gas and ρ_p and d_p are the particle density and diameter respectively.

Similarly for the y-direction
$$U_{py}\frac{dU_{py}}{dy} = \alpha f(U_{gy} - U_{py}). \tag{9}$$

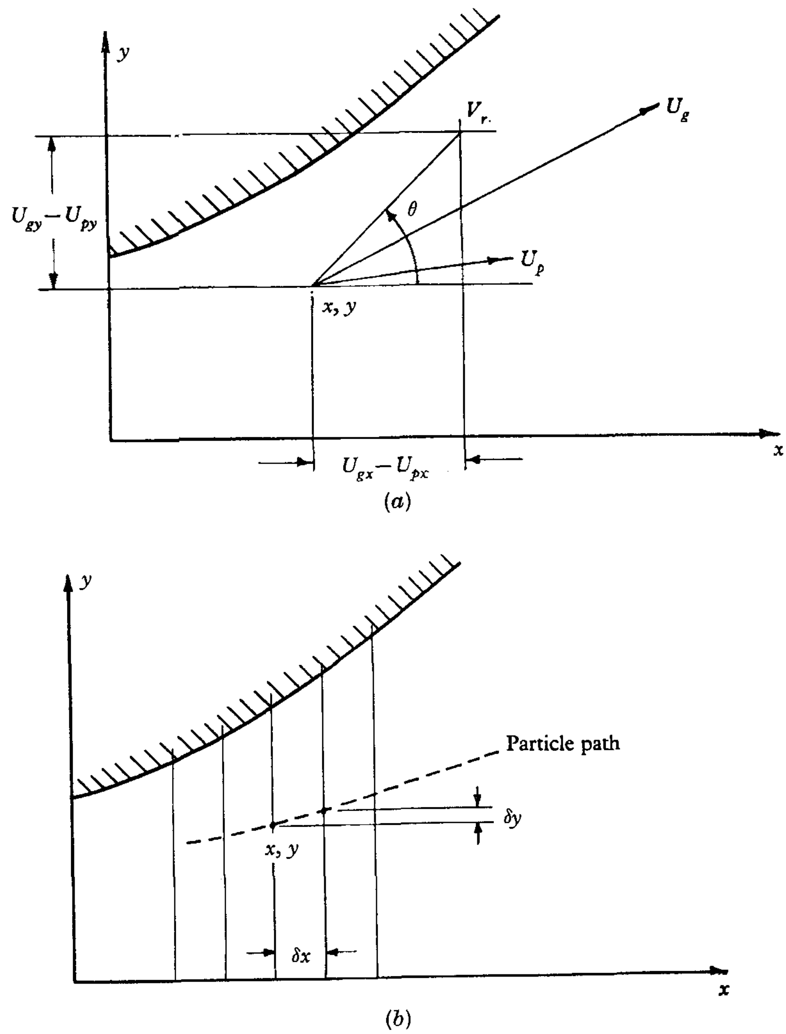


FIGURE 1. Gas and particle velocities, relative velocity and velocity components at a point in a nozzle.

Thus if the particle velocity components U_{px} and U_{py} are known or are assumed at a given x station in the nozzle, the new value of U_{px} at station $x + \delta x$ may be obtained from (7). The time taken by the particle to traverse length δx may then be determined and this time may then be used in conjunction with (9) to determine the distance δy travelled by the particle in the y-direction. Iterative procedures are required for all of these operations before moving on to the next δx interval and so establishing the path taken by the particle.

3. Analytical solutions for supersonic régime trajectories

The method outlined above was applied for supersonic flow in the two-dimensional nozzle shown in figure 2. At the throat section, particles, of density $4000\,kg/m^3$, were assumed to be moving parallel to the nozzle axis with a velocity

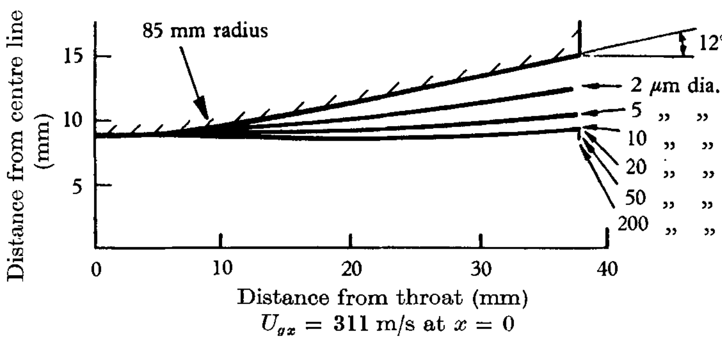


FIGURE 2. Limiting streamlines for particles of different diameters in a divergent nozzle. $\rho_p = 4000\,kg/m^3$; $U_{px} = 244\,m/s$; $U_{py} = 0\,m/s$.

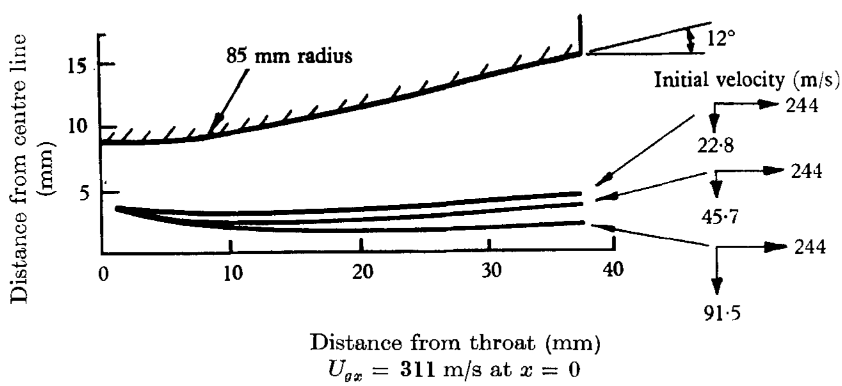


FIGURE 3. Particle streamlines for 2 μm diameter particles showing effects of initial velocity.

of 244 m/s. The limiting particle trajectories (i.e. those corresponding to particles initially at the nozzle wall at the throat section) were obtained for the different particle sizes shown. It will be seen that only very small particles have their position relative to the nozzle centre line appreciably altered as they proceed through the nozzle. A similar result was obtained by Kliegel & Nickerson (1962) for axi-symmetric nozzles.

Figure 3 shows the effect of particles having an initial velocity inclined towards the nozzle centre line. As might be expected, the greater the initial component of velocity normal to the centre line, the greater is the particle concentration around the nozzle axis. For large heavy particles, where the choke geometry gives the particles an appreciable normal component of velocity, it should be possible for

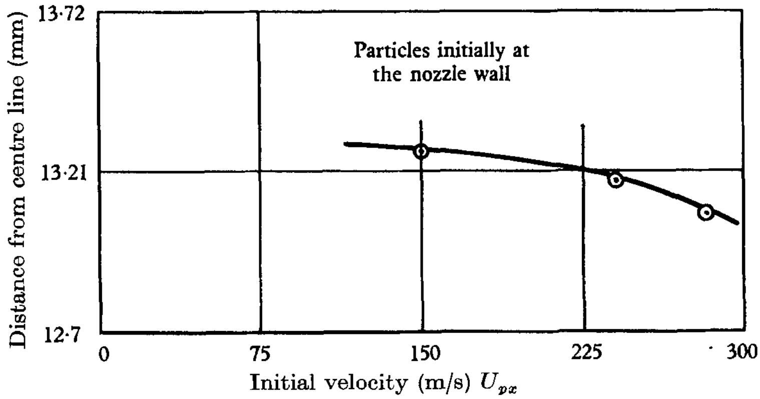


FIGURE 4. Variation in exit position with initial velocity of 2 μm diameter particles at exit of nozzle shown in figure 3. ρ_p = 4000 kg/m³; U_{py} = 0 m/s.

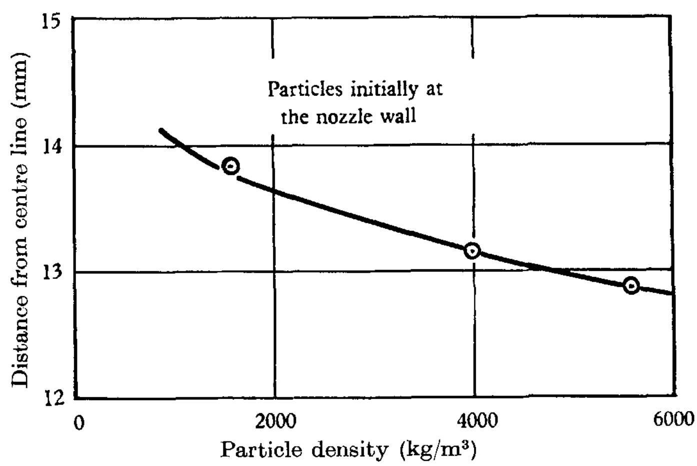


FIGURE 5. Variation in exit position with particle density for 2μm diameter particles at exit of nozzle shown in figure 3. U_{px} = 244 m/s; U_{py} = 0 m/s.

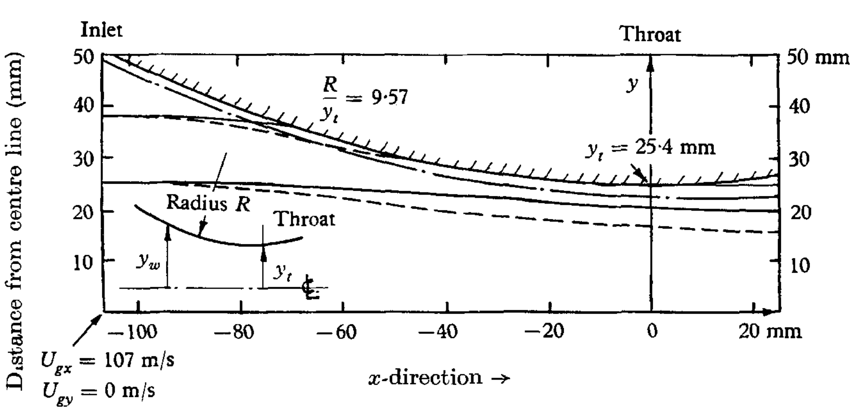
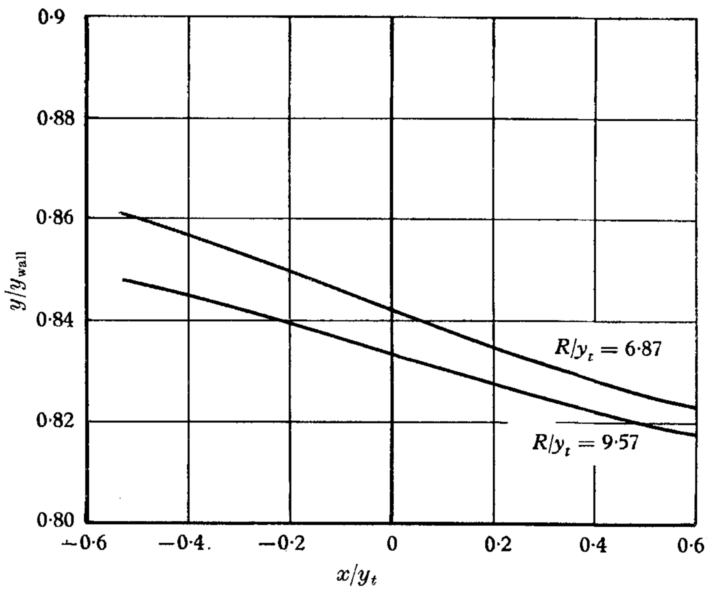


FIGURE 6. Particle trajectories in a subsonic flow régime of a nozzle. U_{px} = 91·5 m/s; U_{py} = 0 m/s; ρ_p = 4000 kg/m³. Particle diameter: —·—, 2 μm; ------, 20 μm; ——, 50μm.

particles to impinge on the opposite nozzle wall. Figures 4 and 5 show the effects of initial particle velocity and of particle density respectively on the position at which the limiting particle trajectory crosses the exit section. From these results it is apparent that the greater the initial particle momentum in a particular direction, the greater is the tendency of the particle to continue in that direction for an appreciable nozzle length.



Particle diam. 2 μm; particle density 4000 kg/m^3

FIGURE 7. Variations of particle position relative to the wall (y/y_{wall}) in the throat regions of two nozzles having different choke radii. Particles initially at the same relative position in nozzles and having the same velocity.

4. Analytical solutions for subsonic régime trajectories

For the choke section shown in figure 6, where the choke radius R is 243·5 mm the resultant particle trajectories for three particle sizes and for different initial positions relative to the nozzle centre line were obtained, and the results are shown in the figure. In all cases the particles were assumed to be travelling parallel to the nozzle centre line and with the same speed at the inlet section. For particles with an initial velocity at entry which, if undisturbed, would cause the particle to strike the wall it is seen that only the smallest particles are so influenced by the air stream that they fail to impinge on the choke wall. Large particles are less influenced by the air stream and are more liable to strike the nozzle wall. The tendency to strike the wall increases as the particle size increases and as the initial distance (in the y-direction) from the nozzle centre line increases.

The effect of decreasing the choke radius from 243·5 to 175 mm is shown in figure 7 for 2 μm particles with the same entry parameters. Here the shorter choke allows the particles to approach the wall more closely. This shows that with a short choke less time is available in which the gas velocity components can influence the particle path. Thus, while one can 'scale' gas velocities in geometrically similar nozzles, particle velocities at similar positions will not be the same since the axial length dimension is an important parameter.

5. Experimental investigations of particle trajectories

A simple experimental technique was used to enable a qualitative comparison to be made between the theoretical results obtained above and the trajectories in actual flows.

In the experiments two-dimensional nozzles were used in which shaped mild steel pieces, forming the nozzle profile, were sandwiched between two Perspex plates. Angular aluminium oxide particles were passed through the nozzles with air as the carrier gas. Scratches on the Perspex walls and mild steel pieces gave an indication of the particle trajectories. A typical result for a convergent–divergent nozzle is shown in figure 8, plate 1.

Experimental results

The effect of different sizes of particle in a convergent–divergent nozzle. Here the nozzles had circular arc chokes with straight diverging walls of semi-angle 7° for the supersonic region. Small quantities of 100, 210 and 500 μm particles were passed through three similar nozzles. The markings on the Perspex and mild steel walls indicated that in the diverging portion the particles tend to congregate in a dense central core and that there was a considerable region around the throat which did not suffer particle attack. The results are illustrated in figure 9. The 'attack free' area is seen to increase with increasing particle size. The theoretical results suggest that the diverging portion should be free from particle attack and that the smaller the particle, the nearer does it approach the nozzle wall. This fits in with the observed pattern of behaviour, but apparently particle–particle interaction also plays a part so that some particles are directed towards the diverging wall.

The effect of different choke lengths. The geometry of the choke profiles used and the results for 210 μm particles are shown in figure 10. The particles approach the diverging wall more closely as the choke length increases. The reason for this is that the more gradual (longer) the choke profile, the more nearly do the particles follow the air stream lines.

The effect of angle of divergence. Three nozzles with different angles of divergence were designed so that the air velocity variation with distance through the nozzle was the same in each. This was achieved by tapering the mild steel pieces.

The angle of divergence and the test results for 210 μm particles are shown in figure 11. It will be seen that the main particle streams are at a considerable distance from the diverging walls except for the nozzle with the smallest angle of divergence. Here the particle stream completely fills the exit cross-section. For the 12° and 20° nozzles a light particle concentration is observed between the main particle stream and the wall, due no doubt to particle–particle interaction. For the 6° nozzle the region of least particle concentration is around the throat area due to the choke causing a vena-contracta effect in the particle stream at this point.

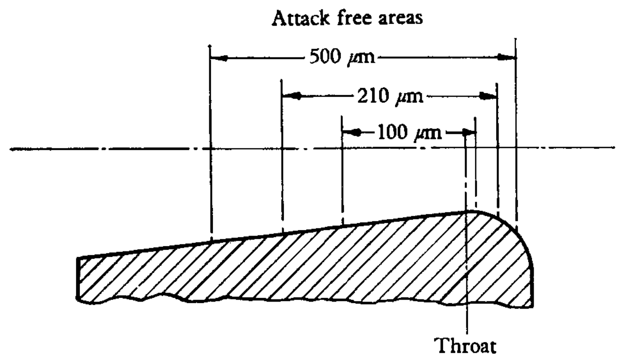


FIGURE 9. Regions of nozzle walls untouched by particles.

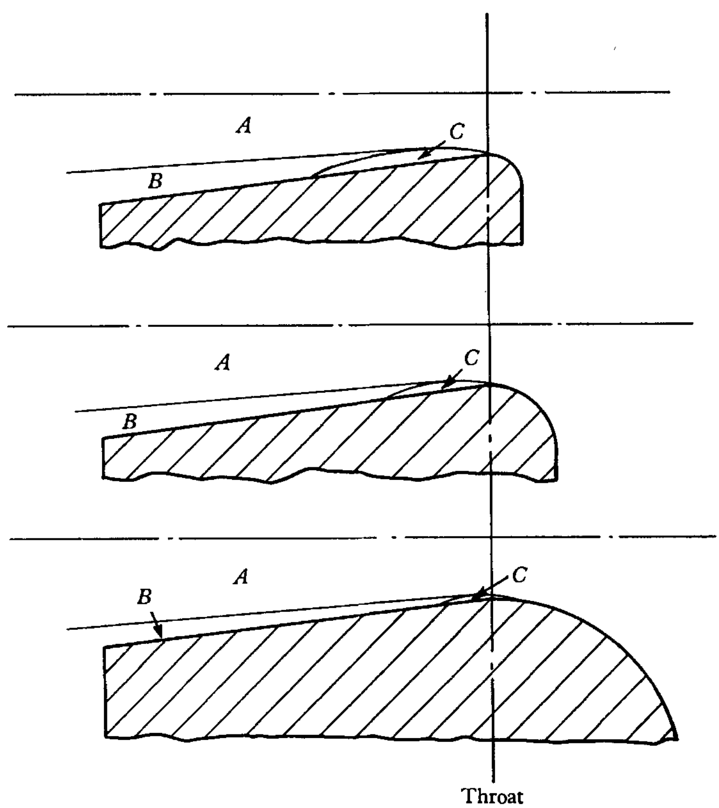


FIGURE 10. Effect of choke radius on particle distribution in a convervent–divergent nozzle. 210 μm. Al Ox particles. Region: A, high particle concentration; B, low particle concentration; C, no particles.

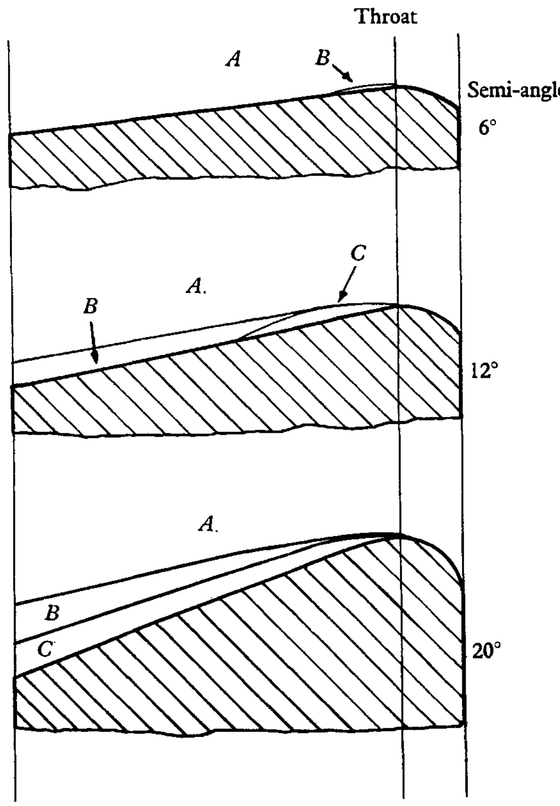


FIGURE 11. Particle trajectories in nozzles having different angles of divergence. 210 μm. Al Ox particles. Region: A, high particle concentration; B, low particle concentration; C, no particles.

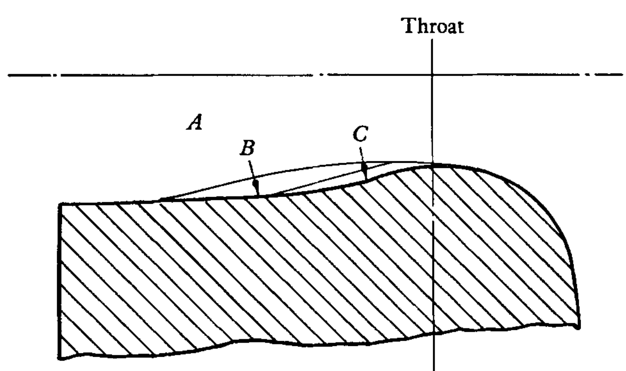


FIGURE 12. Particle concentrations in a parallel flow nozzle. Region: A, high particle concentration; B, low particle concentration; C, no particles.

The effects in a convergent–divergent parallel flow nozzle. In this nozzle the divergent profile was initially convex to the nozzle centre line to give a smooth junction with the throat radius and then concave to the centre line to give parallel flow at exit. The results for 210 μm particles are shown in figure 12. It will be seen that towards the nozzle exit the particle stream spreads to fill the nozzle cross-section and that in the convex region there is only a light concentration of particles. The particles of the main stream strike the profile walls in the concave region. This result is in agreement with calculated results obtained by Bailey, Wilson, Serra & Zuprik (1961) for parallel flow nozzles.

6. Concluding remarks

In the divergent portion of a conical nozzle the trajectories of the particles are influenced by particle size and by particle momentum at the throat section. Ideally if the initial particle velocity is parallel to the nozzle centre line the particles never strike the nozzle wall, though the smaller the particle, the nearer does it approach the wall. The greater the initial particle momentum at the throat, the less is the particle movement towards the wall. In practice, the particles tend to congregate in a dense central core, though particle–particle interaction does influence small particles to move out and attack the nozzle wall. For nozzles designed for parallel flow at exit, particle attack on the concave portion at exit is possible.

Protection of the throat, and areas just downstream of the throat, is obtained if the choke shape is such that at the throat section the particles are given a substantial component of velocity towards the nozzle centre line. This is a vena-contracta effect in the particle flow and increases with decrease in the length of the choke.

Unlike the diverging portion the choke is particularly liable to suffer attack by particles. Again the initial particle momentum at inlet to the choke is an important parameter along with the position of the particle relative to the nozzle centre line. Only the smallest particles of those which are initially directed towards the choke wall will be so influenced by the gas that they fail to hit the nozzle wall at some point in the choke. Long chokes give the drag forces, induced by the relative velocity, more time to act on the particles and hence in long chokes the particles follow the air stream lines more closely.

This work was performed at the University of Strathclyde, Glasgow, under the general direction of Professor A. S. T. Thomson of the Mechanical Engineering Department. The work was initiated by the Ministry of Aviation and Imperial Metal Industries (Kynoch) Ltd., Summerfield Research Station, England.

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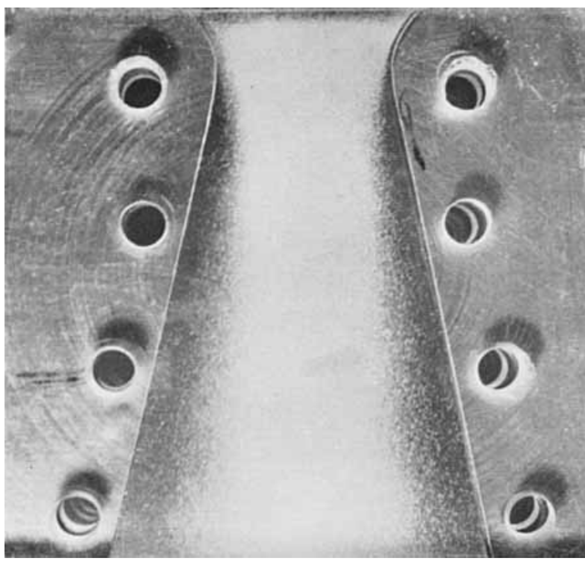

FIGURE 8. Example of marking of Perspex walls.